Quantum metrology applications

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Extreme Light Infrastructure - Nuclear Physics (ELI-NP)

Outlook

Quantum sensing/metrology

A quick look at: market, technologies, industrial applications.

Quantum-enhanced sensing: squeezing metrology

The Shot-noise limit (SNL), the Heisenberg limit and the sub-SNL regime between them.
Coherent states, squeezed states.

Making a case for quantum-enhanced sensing: two case studies

Quantum-enhanced gravitational wave (GW) detection.
Heisenberg-limited microscopy for biological samples.

Table of Contents

Quantum sensing/metrology

Quantum metrology

is the study of making high-resolution and highly sensitive measurements of physical parameters using quantum theory to describe the physical system

– Wikipedia

"Quantum sensing" describes the use of

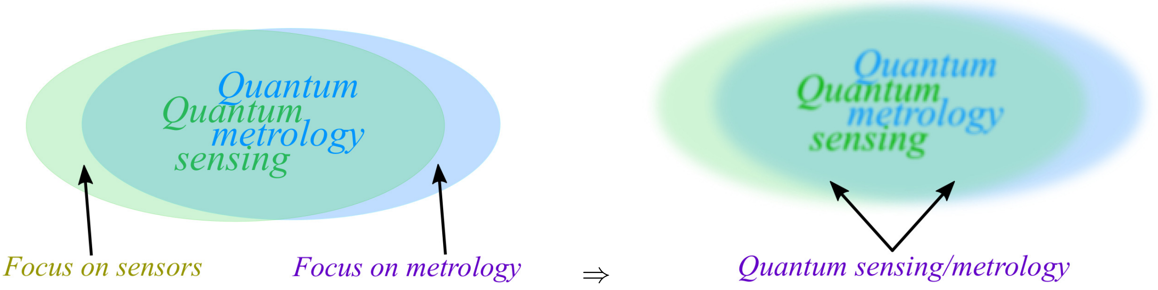
a quantum system, quantum properties, or quantum phenomena to perform a measurement of a physical quantity.

- C. L. Degen, F. Reinhard, and P. Cappellaro *Quantum sensing*, Rev. Mod. Phys.89, 035002 (2017)

Put in another way: if the sensing/measurement process includes such properties as

quantum entanglement, quantum interference, quantum squeezing (and the list can go on) then, yes, what we do is quantum sensing/metrology.

Quantum sensing/metrology



During this presentation, FAPP (for all practical purposes)

sensing = metrology

Quantum sensing market

Quantum sensor market

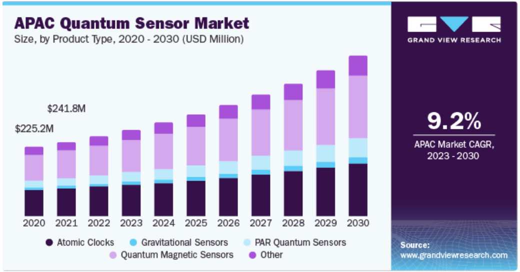
- Global market size value in 2022: 786 M USD
- Global market size value in 2023: 839 M USD
- Expected to grow ~ 7 % per year
- Expected global market 1-6 B USD by 2040 (McKinsey)

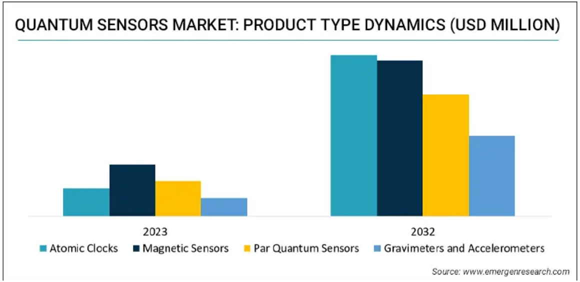
Source: *Quantum Sensor Market Size & Trends*, https://www.grandviewresearch.com/industry-analysis/quantum-sensor-market-report

Note: not all sources agree on the market size, however they seem to agree on the trend, see:

Quantum Sensor Market Size, https://www.gminsights.com/industry-analysis/quantum-sensors-market
Quantum Technology Monitor, Quantum Technology Monitor - McKinsey & Company
Quantum Sensors Market Poised for Explosive Growth, https://finance.yahoo.com/news/quantum-sensors-market-poised-explosive-110500729.html

Quantum sensing market





The main quantum sensors by type:

- Atomic clocks
- Photosynthetically Active Radiation (PAR) quantum sensors
- Quantum magnetic sensors
- Gravimeters and Accelerometers

Sources:
1) *Quantum Sensor Market Size & Trends*, https://www.grandviewresearch.com/industry-analysis/quantum-sensor-market-report
2) *Quantum Sensor Industry Overview*, https://www.emergenresearch.com/industry-report/quantum-sensors-market

Quantum sensing by implementation

TABLE I. Experimental implementations of quantum sensors.

Implementation	Qubit(s)	Measured quantity(ies)	Typical frequency	Initalization	Readout	Type[a]
Neutral atoms						
Atomic vapor	Atomic spin	Magnetic field, rotation, time/frequency	dc-GHz	Optical	Optical	II, III
Cold clouds	Atomic spin	Magnetic field, acceleration, time/frequency	dc-GHz	Optical	Optical	II, III
Trapped ion(s)						
	Long-lived electronic state	Time/frequency	THz	Optical	Optical	II, III
		Rotation		Optical	Optical	II
	Vibrational mode	Electric field, force	MHz	Optical	Optical	II
Rydberg atoms						
	Rydberg states	Electric field	dc, GHz	Optical	Optical	II, III
Solid-state spins (ensembles)						
NMR sensors	Nuclear spins	Magnetic field	dc	Thermal	Pick-up coil	II
NV[b] center ensembles	Electron spins	Magnetic field, electric field, temperature, pressure, rotation	dc-GHz	Optical	Optical	II
Solid-state spins (single spins)						
P donor in Si	Electron spin	Magnetic field	dc-GHz	Thermal	Electrical	II
Semiconductor quantum dots	Electron spin	Magnetic field, electric field	dc-GHz	Electrical, optical	Electrical, optical	I, II
Single NV[b] center	Electron spin	Magnetic field, electric field, temperature, pressure, rotation	dc-GHz	Optical	Optical	II
Superconducting circuits						
SQUID[c]	Supercurrent	Magnetic field	dc-GHz	Thermal	Electrical	I, II
Flux qubit	Circulating currents	Magnetic field	dc-GHz	Thermal	Electrical	II
Charge qubit	Charge eigenstates	Electric field	dc-GHz	Thermal	Electrical	II
Elementary particles						
Muon	Muonic spin	Magnetic field	dc	Radioactive decay	Radioactive decay	II
Neutron	Nuclear spin	Magnetic field, phonon density, gravity	dc	Bragg scattering	Bragg scattering	II
Other sensors						
SET[d]	Charge eigenstates	Electric field	dc-MHz	Thermal	Electrical	I
Optomechanics	Phonons	Force, acceleration, mass, magnetic field, voltage	kHz–GHz	Thermal	Optical	I
Interferometer	Photons, (atoms, molecules)	Displacement, refractive index	...			II, III

[a]Sensor type refers to the three definitions of quantum sensing in Sec. II.A.
[b]*NV*: nitrogen vacancy.

Source: C. L. Degen, F. Reinhard, and P. Cappellaro *Quantum sensing*, Rev. Mod. Phys. 89, 035002 (2017)

Quantum sensing by implementation

The field is very broad,

ranging from elementary particles (*e. g.* muons, "muon spin rotation" (μSR), see reference below) to NV centers [featuring sensitivities 250 aT/($\sqrt{\text{Hz}}$/cm$^{-3/2}$)] and SQUIDs (superconducting quantum interference devices), to mention just a few.

In the following I will focus on photonic devices

and detail some applications (LIGO/Virgo, Heisenberg-limited microscopy for biological samples) and techniques used (squeezing).

Jeff E. Sonier, Jess H. Brewer, and Robert F. Kiefl, *μSR studies of the vortex state in type-II superconductors*, Rev. Mod. Phys. 72, 769 (2000)
J. M. Taylor *et al.*, *High-sensitivity diamond magnetometer with nanoscale resolution*, Nat. Phys. 4, 810 (2008)
M. Simmonds, W. Fertig and R. Giffard, *Performance of a resonant input SQUID amplifier system*, IEEE Transactions on Magnetics, 15, 478 (1979)

Table of Contents

The shot-noise limit and Heisenberg limit

Fact:

The average number of input photons ($\bar{N}$) is **the resource you have**.

Note: It is proportional to your energy, so to the cost you put into your experiments.

So how much "sensitivity per buck" can you hope for?

The classical limit

Also called the shot-noise limit (SNL) or the standard quantum limit (SQL) is given by

$$\Delta\varphi_{SNL} = \frac{1}{\sqrt{\bar{N}}}$$

The Heisenberg limit (HL)

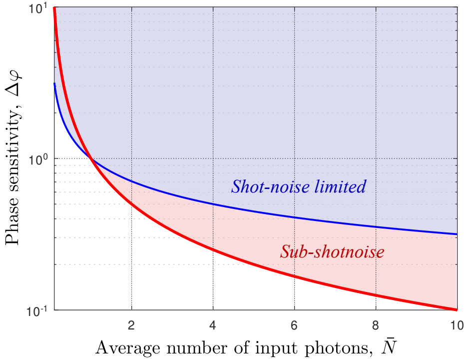
is the ultimate quantum limit and it is given by:

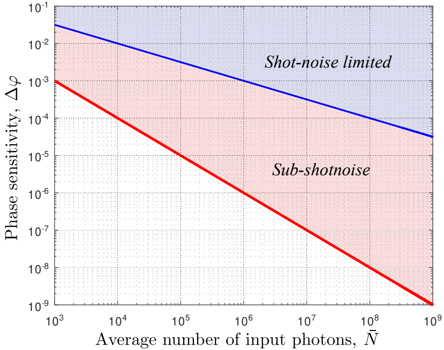
$$\Delta\varphi_{HL} = \frac{1}{\bar{N}}$$

- Giovannetti, Lloyd & Maccone, *Quantum-Enhanced Measurements: Beating the Standard Quantum Limit*, Science 306 1330 (2004)
- Giovannetti, Lloyd & Maccone, *Quantum Metrology*, Phys. Rev. Lett. 96, 010401 (2006)
- Demkowicz-Dobrzański, Jarzyna, and Kołodyński, *Quantum Limits in Optical Interferometry*, Progress in Optics, 60, 345 (2015)

The shot-noise limit and Heisenberg limit

$$\Delta\varphi_{SNL} = \frac{1}{\sqrt{\bar{N}}} \text{ (blue curve)} \qquad \Delta\varphi_{HL} = \frac{1}{\bar{N}} \text{ (red curve)}$$



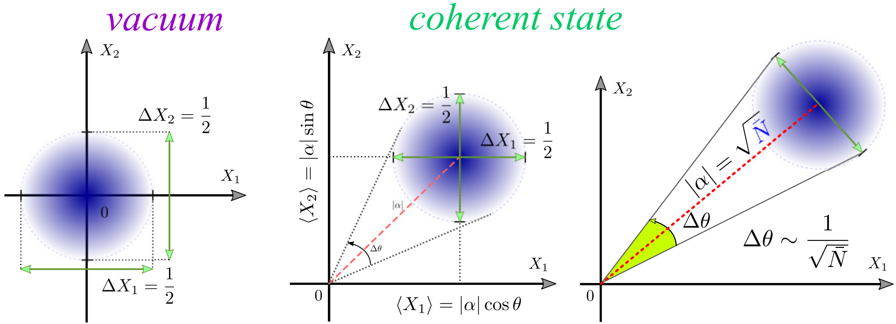


Remark: for $\bar{N} \gg 1$ there is plenty of space between the SNL and the HL:

$$\frac{1}{\bar{N}} = \Delta\varphi_{HL} \leq \Delta\varphi \leq \Delta\varphi_{SNL} = \frac{1}{\sqrt{\bar{N}}}$$

The coherent states – "the most classical states" in quantum optics

The coherent states can be viewed as a displacement of the quantum vacuum state.
(So a laser is just a displaced vacuum state.)



Fact:

Irrespective on the value of $|\alpha|$ (i.e. the coherent state intensity) we have the variances on both quadratures equal and identical to the variance of the vacuum state.

Squeezed states of light – squeezed vacuum

Fact:

Heisenberg's uncertainty principle requires that

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

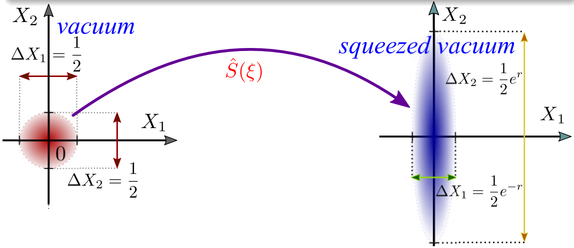
however, it does not put constraints on ΔX_1 (or ΔX_2) separately.

Another fact:

if we have a (quantum) state of light with

$$\Delta X_1 = \frac{1}{2}e^{-r} \quad \text{and} \quad \Delta X_2 = \frac{1}{2}e^{r}$$

Heisenberg would not object to it.



We still have $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$.

The principle of squeezing:

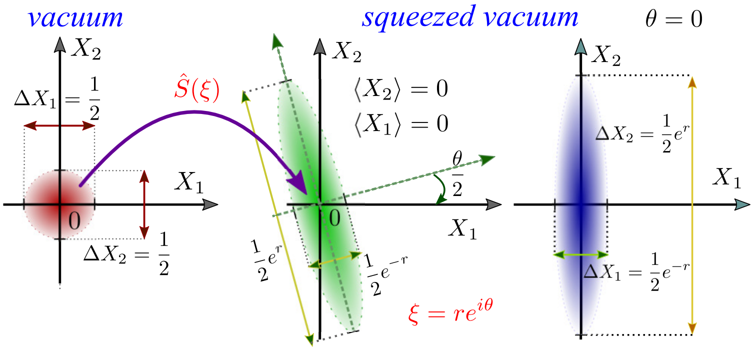
minimize the fluctuations on one axis (the one of interest).

Squeezed states of light – quantum optically speaking

The squeezed vacuum state is given by $|\xi\rangle = \underbrace{\hat{S}(\xi)}_{\text{squeezes the vacuum}} |0\rangle$

where $\xi = re^{i\theta}$ and r is the squeezing factor.
The squeezing operator is

$$\hat{S}(\xi) = e^{\frac{1}{2}\left[\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2}\right]}$$



The average number of photons

(i. e. the squeezed vacuum has photons)

$$\langle \hat{n} \rangle = \sinh^2 r = \bar{N}$$

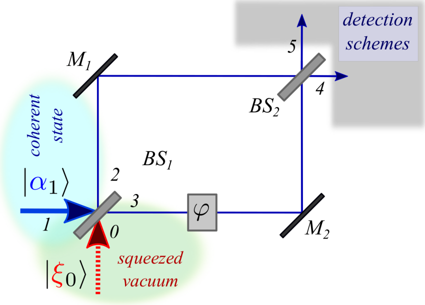
$$|\xi\rangle \sim |0\rangle + c_2|2\rangle + c_4|4\rangle + \dots$$

The coherent plus squeezed vacuum (CSV) input state

The input state of coherent plus squeezed vacuum

$$|\psi_{in}\rangle = |\alpha_1 \xi_0\rangle = \underbrace{\hat{D}_1(\alpha)}_{\text{displaces the vaccum}} \underbrace{\hat{S}_0(\xi)}_{\text{squeezes the vacuum}} |0\rangle$$

is widely used.



Why this state?

Because we can potentially go below the SNL towards the elusive Heisenberg limit:

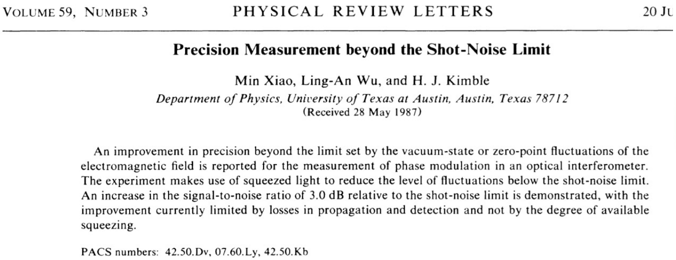
$$\frac{1}{\sqrt{\bar{N}}} \geq \Delta\varphi_{CSV} \geq \frac{1}{\bar{N}}$$

CSV for sub-shot noise sensitivity

The credit for this idea...

... goes to Carlton Caves (see reference below).

This assertion is then experimentally proven:

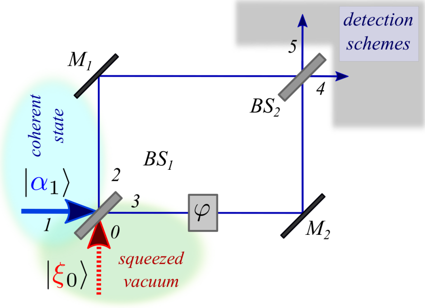

VOLUME 59, NUMBER 3 PHYSICAL REVIEW LETTERS 20 JU

Precision Measurement beyond the Shot-Noise Limit

Min Xiao, Ling-An Wu, and H. J. Kimble
Department of Physics, University of Texas at Austin, Austin, Texas 78712
(Received 28 May 1987)

An improvement in precision beyond the limit set by the vacuum-state or zero-point fluctuations of the electromagnetic field is reported for the measurement of phase modulation in an optical interferometer. The experiment makes use of squeezed light to reduce the level of fluctuations below the shot-noise limit. An increase in the signal-to-noise ratio of 3.0 dB relative to the shot-noise limit is demonstrated, with the improvement currently limited by losses in propagation and detection and not by the degree of available squeezing.

PACS numbers: 42.50.Dv, 07.60.Ly, 42.50.Kb



- Carlton M. Caves, *Quantum-mechanical noise in an interferometer*, Phys. Rev. D 23, 1693 (1981)
- M. Xiao, L.-A. Wu, and J. Kimble, *Precision measurement beyond the shot-noise limit*, Phys. Rev. Lett. 59, 278 (1987)

Sub-shot noise sensitivity - experimental proof (1987)

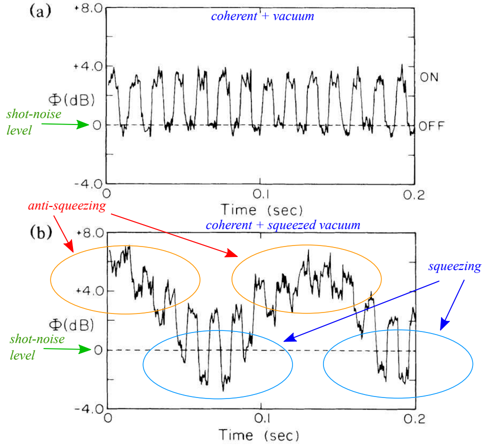


Image from M. Xiao, L.-A. Wu, and J. Kimble, *Precision measurement beyond the shot-noise limit*, Phys. Rev. Lett. 59, 278 (1987)

Sub-shot noise sensitivity for GW detection (2010)

More recent squeezing for gravitational wave (GW) astronomy:

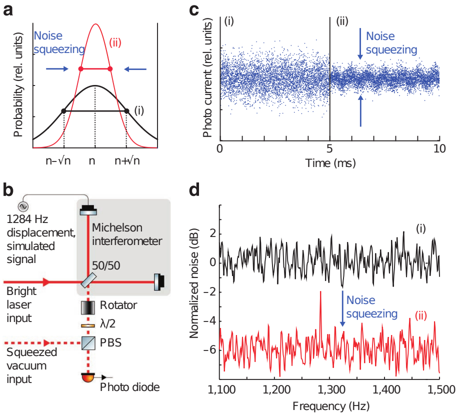
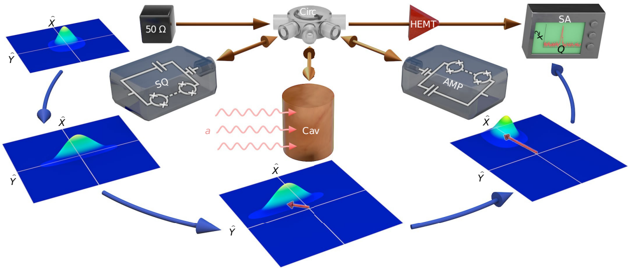


Image credit: R. Schnabel et al., *Quantum metrology for gravitational wave astronomy*, Nature Communications 1, 121 (2010)

Quantum enhanced searches

"Here we use vacuum squeezing to circumvent the quantum limit in a search for dark matter."



"By preparing a microwave-frequency electromagnetic field in a squeezed state and near-noiselessly reading out only the squeezed quadrature, we double the search rate for axions over a mass range favoured by some recent theoretical projections"

Backes *et al.*, A quantum enhanced search for dark matter axions, Nature 590 238 (2021)

Squeezing enhanced metrology – conclusions

Applications:

- **gravitational wave astronomy** (LIGO, VIRGO, GEO600, KAGRA)
- quantum-enhanced BSM particle searches
- quantum amplification of mechanical motion
- biological measurements

- LIGO Scientific Collaboration, *Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light*, Nature Photonics 7, 613 (2013) | arXiv:1310.0383 [quant-ph]
- VIRGO Scientific Collaboration, *Advanced Virgo: a second-generation interferometric gravitational wave detector*, Classical and Quantum Gravity 32, 024001 (2014) | arXiv:1408.3978 [gr-qc]
- M. Tse et al., *Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy*, Phys. Rev. Lett. 123, 231107 (2019)
- M. Taylor et al., *Biological measurement beyond the quantum limit*, Nat. Photon. 7, 229 (2013) | arXiv:1206.6928 [quant-ph]
- S. Burd et al., *Quantum amplification of mechanical oscillator motion*, Science 364, 1163 (2019) | arXiv:1812.01812 [quant-ph]

N00N states' super-resolution - experimental results

The NOON states:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\mathrm{N}\rangle + |\mathrm{N}0\rangle\right)$$

the most well-known being

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|02\rangle + |20\rangle\right).$$

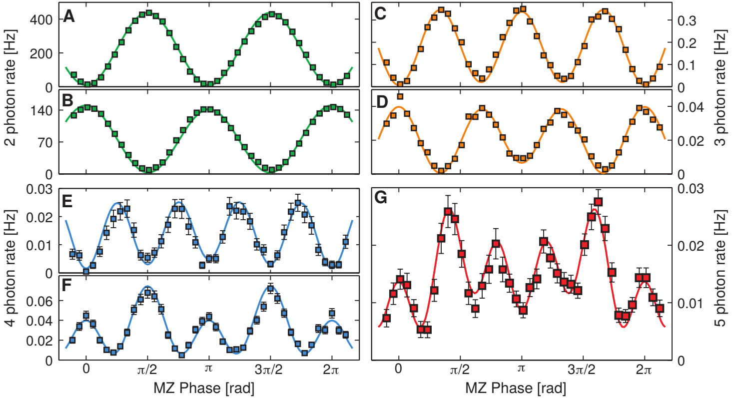


Image credit: Itai Afek, Oron Ambar and Yaron Silberberg

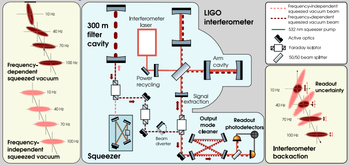
- Itai Afek, Oron Ambar, Yaron Silberberg, *High-NOON States by Mixing Quantum and Classical Light*, Science 328, 879 (2010)

Table of Contents

Quantum sensing: two case studies

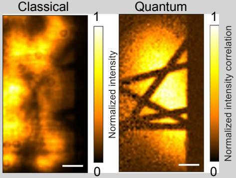
Quantum-enhanced GW (gravitational waves) detection

Sub-SNL sensitivity by employing the squeezed states

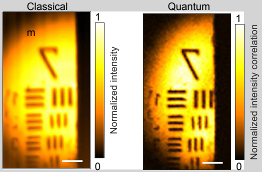


Quantum-enhanced microscopy

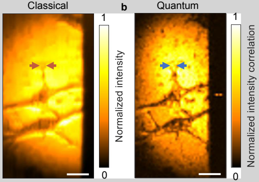
Use entangled NOON states in order to image fragile biological samples (*e. g.* live cells).



Classical and QMC images of carbon fibers in the presence of stray light

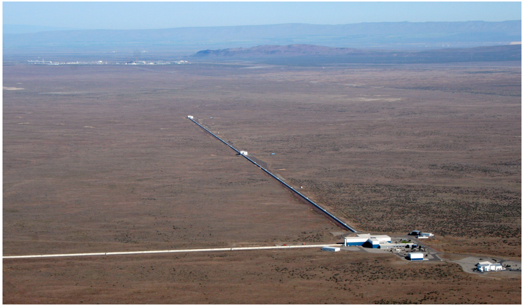


Spatial resolution of QMC



Imaging of cancer cells with QMC

LIGO (4km arms) and Virgo (3km arms): largest Michelson interferometers



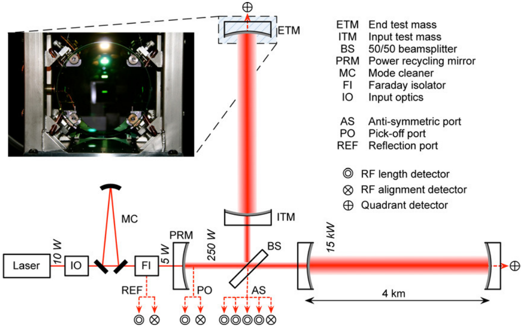
LIGO Hanford, Washington, USA

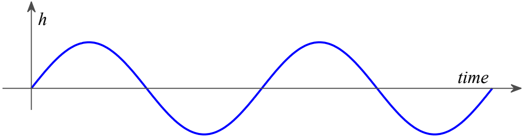
Image courtesy: LIGO Scientific Collaboration

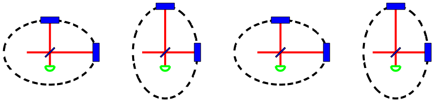
Virgo, Cascina, Italy

Image courtesy: EGO

LIGO GW (graviational wave) interferometer







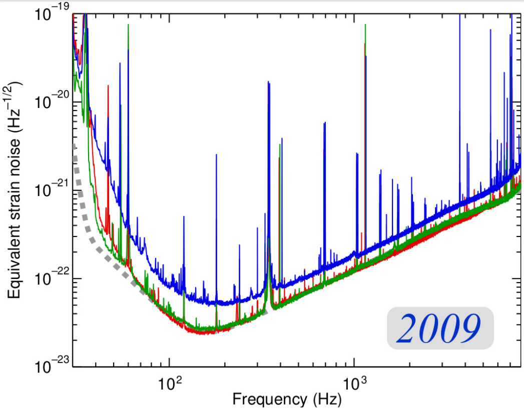
Michelson-type interferometer: the ideal shot-noise limited strain noise density

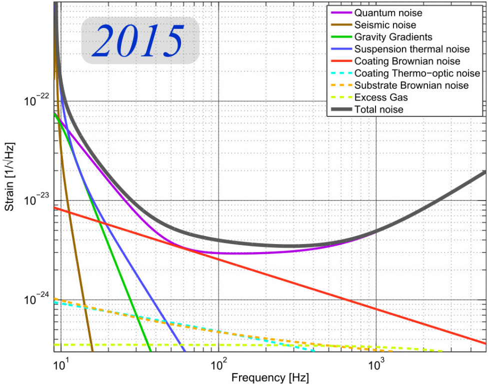
$$\tilde{h}(f) = \sqrt{\frac{\pi \hbar \lambda}{\eta P_{BS} c}} \frac{\sqrt{1 + (4\pi f \tau_s)^2}}{4\pi \tau_s}$$

- λ – laser wavelength
- f – GW frequency
- c – speed of light
- τ_s – arm cavity storage time
- P_{BS} – power incident on the BS
- η – photodetector efficiency

Image source: Abbott *et al.*, *LIGO: the Laser Interferometer Gravitational-Wave Observatory*, Rep. Prog. Phys. 72 076901 (2009)

LIGO with classical input light





Michelson-type interferometer: the ideal shot-noise limited strain noise density

target: $\tilde{h}\,(f = 100\;Hz) = 1.0 \times 10^{-23}\text{Hz}^{-1/2}$

actual: $\tilde{h}\,(f = 100\;Hz) = 1.3 \times 10^{-23}\text{Hz}^{-1/2}$

$$\begin{cases} \tau_s P_{BS} = 0.9 \times 250\text{ W} \\ \tau_s = 1\text{ ms} \end{cases}$$

LIGO with squeezed light

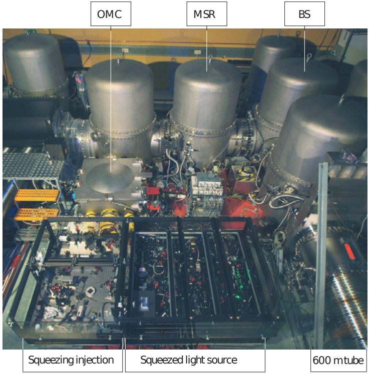


Figure 2 | View into the GEO 600 central building. In the front, the squeezing bench containing the squeezed-light source and the squeezing injection path is shown. The optical table is surrounded by several vacuum chambers containing suspended interferometer optics.

With contributions from many groups around the world (GEO600 notably), and following Caves' 1981 proposal, work on a squeezed vacuum source progresses.

LETTERS
PUBLISHED ONLINE: 11 SEPTEMBER 2011 | DOI: 10.1038/NPHYS2083
nature physics

A gravitational wave observatory operating beyond the quantum shot-noise limit

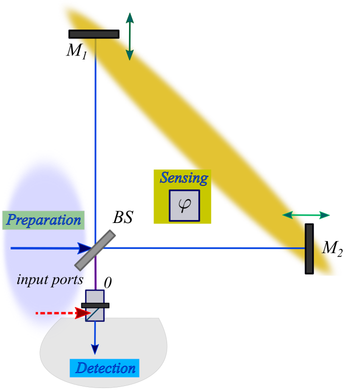
The LIGO Scientific Collaboration †*

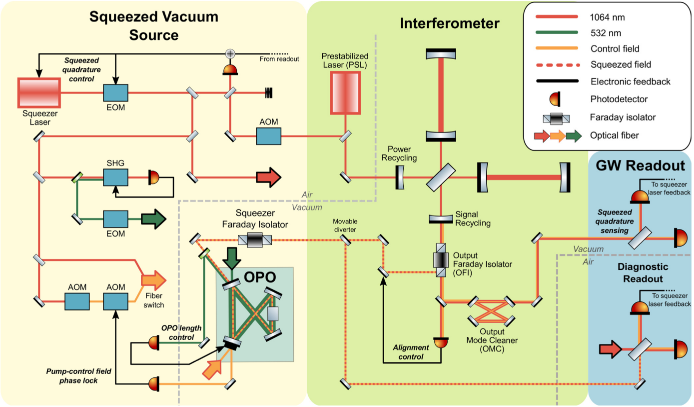
nature photonics
LETTERS
PUBLISHED ONLINE: 21 JULY 2013 | DOI: 10.1038/NPHOTON.2013.177

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

GW interferometer with squeezed vacuum input



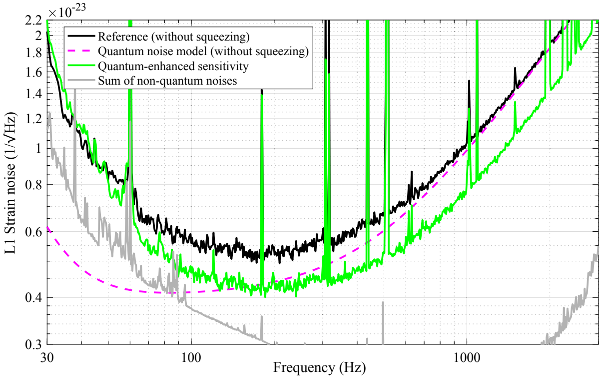


Note: being a Michelson type interferometer,

the squeezed vacuum is injected from the output port via a Faraday isolator.

Right image from: M. Tse et al., Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy, Phys. Rev. Lett. 123, 231107 (2019)

LIGO sub-shot noise sensitivity for GW detection (2019)



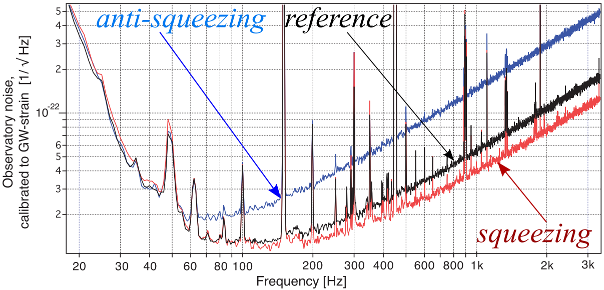
"We note that to achieve the same reduction in shot noise would require 85% (L1) and 65% (H1) more laser power, which is beyond the capability of the current laser system."

"Moreover, increasing the laser power complicates the control of the interferometer due to thermal effects, angular instabilities caused by photon radiation-pressure induced torques, and parametric instabilities"

Squeezing ≈ 2.7 dB.

M. Tse et al., Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy, Phys. Rev. Lett. 123, 231107 (2019)

Virgo sub-shot noise sensitivity for GW detection (2019)



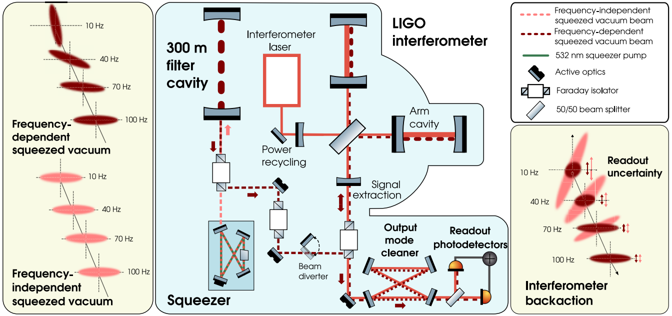
"Since up to now the quantum radiation pressure noise is just below the residual technical noise sources at low GW detection frequencies, a broadband sensitivity improvement can be achieved by reducing the shot noise contribution via a moderate injection of frequency- independent squeezed vacuum states, whose fluctuations are reduced in the quadrature of the light aligned with the gravitational-wave signal."

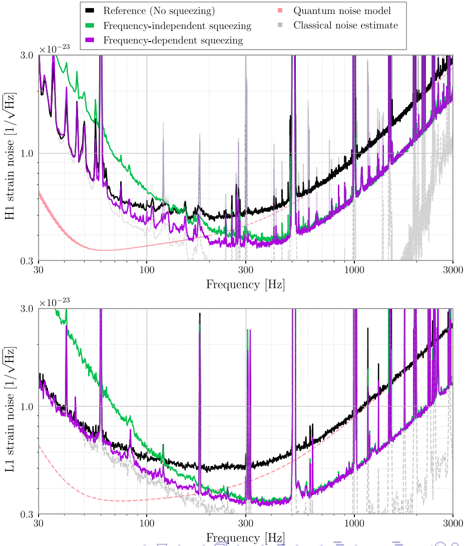
Sensitivity ≈ 3.2 dB below shot-noise.
(Squeezing injected ≈ 10 dB).

F. Acernese et al., Increasing the Astrophysical Reach of the Advanced Virgo Detector via the Application of Squeezed Vacuum States of Light, Phys. Rev. Lett. 123, 231108 (2019)

GW enhanced sensitivity with frequency-dependent squeezing (2023)

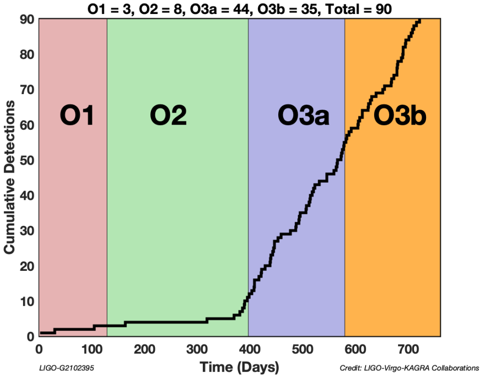
As if squeezing was not good enough, it recently **became even better**:

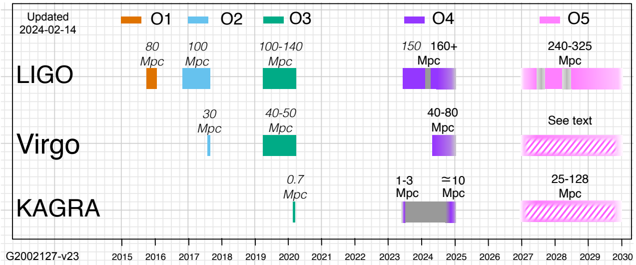




Images:
D. Ganapathy et al. *Broadband Quantum Enhancement of the LIGO Detectors with Frequency-Dependent Squeezing*, Phys. Rev. X 13, 041021 (2023)

GW astronomy got a quantum enhancement





Detector Status Portal:
https://online.igwn.org

What caused the rapid increase in **detected events**? Of course, the *squeezed vacuum*.

GW quantum-boosted sensitivity

The GW interferometers are

classically optimized at the SQL (standard quantum limit).

The SQL is the result of a compromise

between the radiation pressure noise (directly proportional to the optical power impinging on the test masses and inversely proportional to the square of the Fourier frequency), and the shot noise (inversely proportional to the operating optical power).

Quantum-enhanced interferometry

uses squeezing to beat the SQL.

Classical light

implies $\sim \mathbf{10^{19}}$ photons/ms in the GW interferometer cavities.

The quantum advantage comes from only

$\sim \mathbf{2}$ photons/ms injected into the output port.

Quantum advantage example: Heisenberg-limited microscopy

Quantum-enhanced microscopy

There is a clear interest to lower as much as possible the exposure to light of biological samples during imaging.

Sometimes the classical limit

is simply too "bright" for the sensitive sample. (Live cell samples, retina cells.)

Quantum-enhanced microscopy can lower this limit

How? Using non-classical properties of light.

Andrew G. White, Jay R. Mitchell, Olaf Nairz, and Paul G. Kwiat, "Interaction-free" imaging, Phys. Rev. A 58, 605 (1998)
Paul G. Kwiat, Experimental and theoretical progress in interaction-free measurements, Phys. Scr. 1998 115 (1998)
Mitchell, M., Lundeen, J. and Steinberg, A. M., Super-resolving phase measurements with a multiphoton entangled state, Nature 429, 161 (2004)
Michael A. Taylor *et al.*, Subdiffraction-Limited Quantum Imaging within a Living Cell, Phys. Rev. X 4, 011017 (2014)
Moreau, PA., Toninelli, E., Gregory, T. *et al.*, Imaging with quantum states of light, Nat Rev Phys 1, 367 (2019)
Z. He *et al.*, Quantum microscopy of cells at the Heisenberg limit, Nat Commun 14, 2441 (2023)

Quantum imaging digression - types of quantum imaging

Ghost imaging

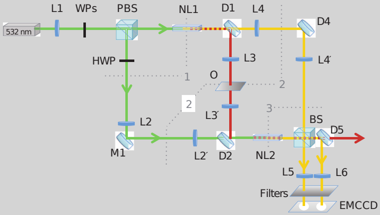
Use correlated photon pairs and image an object with a photon that never interacted with it.

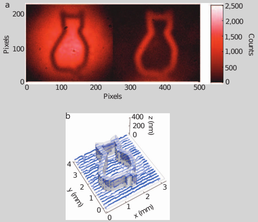
Miles J. Padgett and Robert W. Boyd, An introduction to ghost imaging: quantum and classical, Phil. Trans. R. Soc. A. 375 20160233 (2017)

Hance, J.R., Rarity, J, Counterfactual ghost imaging, npj Quantum Inf, 7, 88 (2021).

Quantum imaging with undetected photons

Use induced coherence in order to image an object with a photon that never interacted with it.





Lemos, G. et al., Quantum imaging with undetected photons, Nature 512, 409 (2014).

Quantum imaging digression - types of quantum imaging (continued)

Interaction-free imaging

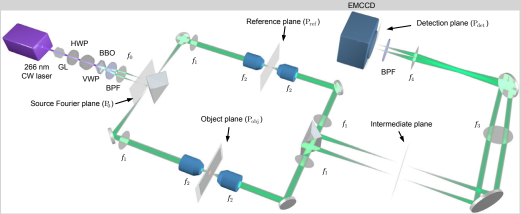
Use an Elitzur-Vaidman type setup to infer photons who actually did not reach the sample.

Andrew G. White, Jay R. Mitchell, Olaf Nairz, and Paul G. Kwiat, "Interaction-free" imaging, Phys. Rev. A 58, 605 (1998)
Paul G. Kwiat, Experimental and theoretical progress in interaction-free measurements, Phys. Scr. 1998 115 (1998)

A. M. Pălici, T.-A. Isdrailă, S. Ataman, and R. Ionicioiu, Interaction-free imaging of multipixel objects, Phys. Rev. A 105, 013529 (2022)

Entanglement-enhanced quantum imaging

Use entangled NOON states to achieve the Heigenberg scaling.

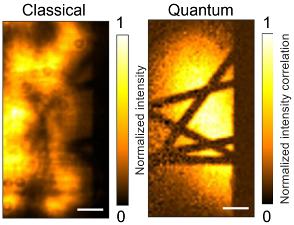


Mitchell, M., Lundeen, J. and Steinberg, A. M., Super-resolving phase measurements with a multiphoton entangled state, Nature 429, 161 (2004)
Z. He et al., Quantum microscopy of cells at the Heisenberg limit, Nat Commun 14, 2441 (2023)

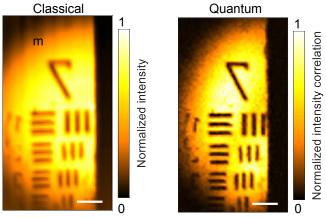
Quantum microscopy

Entangled biphoton-based quantum imaging

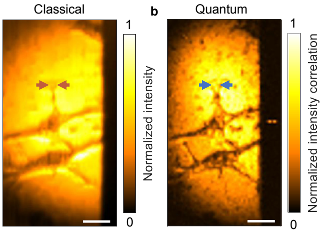
"Here, we present quantum microscopy by coincidence (QMC) with balanced pathlengths, which enables super-resolution imaging at the Heisenberg limit with substantially higher speedsand CNRs (contrast-to-noise ratios) than existing wide-field quantum imaging methods."



Classical and QMC images of carbon fibers in the presence of stray light



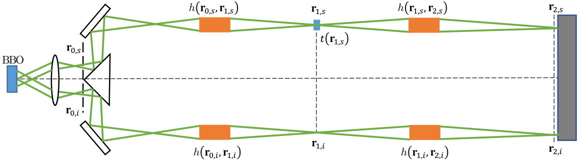
Spatial resolution of QMC



Imaging of cancer cells with QMC

Images: Z. He et al., Quantum microscopy of cells at the Heisenberg limit, Nat Commun 14, 2441 (2023).

Quantum microscopy



"In conclusion, we have demonstrated quantum microscopy of cancer cells at the Heisenberg limit. QMC is advantageous over existing wide-field quantum imaging methods due to the 1.4 μm resolution, up to 5 times higher speed, 2.6 times higher CNR, and 10 times more robustness to stray light. With low-intensity illumination, we have demonstrated that QMC is suitable for nondestructive bioimaging at a cellular level, revealing details that cannot be resolved by its classical counterpart."

Images: Z. He et al., Quantum microscopy of cells at the Heisenberg limit, Nat Commun 14, 2441 (2023).

Table of Contents

Conclusions

Quantum sensing/metrology

Is an industry and we are already in the implementation phase.
Research in quantum metrology is still a (very) active field.

Quantum-enhanced metrology

Is a sensitivity booster for both high and low intensities.
Squeezing is a technique of paramount importance. Obtaining higher squeezing factors is an ongoing experimental endeavour.

Quantum sensing/metrology

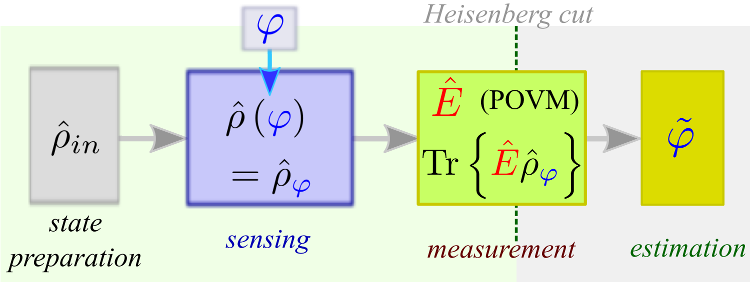
is one of the pillars of (the still developing) quantum technologies.

That's all, folks!

Thank you for your attention!

Questions are welcome.

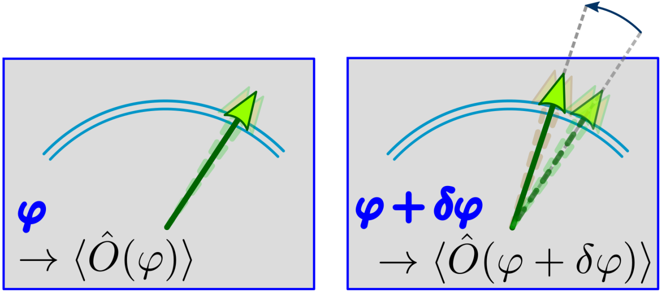
General parameter estimation - stating the problem



The observables

In the sensing process we go from $\hat{\rho}_{in}$ (or $|\psi_{in}\rangle$ for pure states) to $\hat{\rho}_\varphi = \hat{U}^\dagger(\varphi)\,\hat{\rho}_{in}\hat{U}(\varphi)$ and usually $\hat{U}(\varphi) = e^{i\varphi\hat{O}}$.

Parameter estimation: how sensitive can we be?



A *small* variation $\delta\varphi$ of the parameter φ induces a change in the observable i. e.

$$\langle \hat{O}\left(\varphi\right) \rangle \to \langle \hat{O}\left(\varphi + \delta\varphi\right) \rangle$$

(Legit) question: what is the smallest value of $\delta\varphi$

that still yields detectable results via measurements on the operator $\hat{O}\left(\varphi\right)$?

Parameter estimation – the error propagation formula

(Phase) sensitivity - $\Delta\varphi$

The value of $\delta\varphi$ that *saturates* the above inequality is called *sensitivity*,

$$\Delta\varphi = \frac{\sqrt{\Delta^2\hat{O}(\varphi)}}{\left|\frac{\partial}{\partial\varphi}\langle\hat{O}\rangle\right|} \quad \text{or simply} \quad \boxed{\Delta\varphi = \frac{\Delta\hat{O}}{\left|\frac{\partial}{\partial\varphi}\langle\hat{O}\rangle\right|}}$$

and this is the famous *error propagation formula*.

This equation implies that we know the operator $\hat{O}$.
However, $\hat{O}$ might not be optimal.
Question: could we give a best-case scenario for $\Delta\varphi$ over all imaginable operators $\hat{O}$?

Coherent plus squeezed vacuum: the resource we want to optimize

Input state:

$$|\psi_{in}\rangle = |\alpha_1 \xi_0\rangle = \hat{D}_1(\alpha)\,\hat{S}_0(\xi)\,|0\rangle \ \text{ with } 2\theta_\alpha - \theta = 0.$$

Our resource:

is the average number of input photons $\bar{N}$.

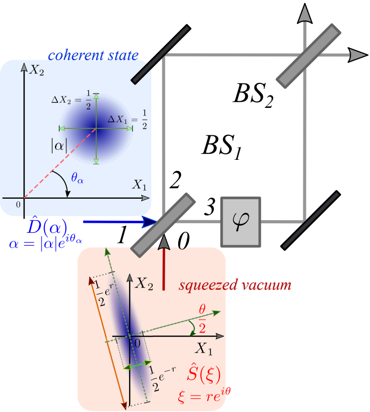
For a coherent plus squeezed vacuum input we have $\bar{N} = |\alpha|^2 + \sinh^2 r$.

Define the coherent fraction:

$$f_\alpha = \frac{|\alpha|^2}{\bar{N}}.$$

Question:

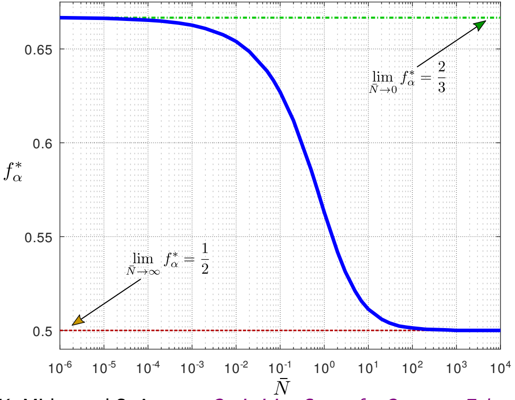
What value f_α maximizes the performance in terms of QFI?



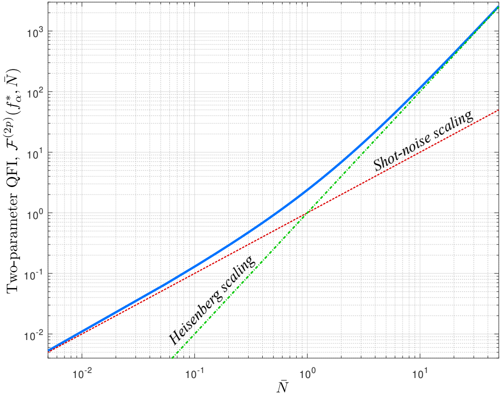
• K. Mishra and S. Ataman, *Optimizing States for Quantum-Enhanced Interferometry: Two Case Studies*, LPHYS'23 - to appear soon in the proceedings (2024)

Optimal two-parameter QFI for a coherent + squeezed vacuum input

The optimum f_α in respect with the total average number of input photons.



The f_α-optimized two-parameter QFI $\mathcal{F}^{(2p)}(f_\alpha^*, \bar{N})$ versus the average input photon number $\bar{N}$.



- K. Mishra and S. Ataman, *Optimizing States for Quantum-Enhanced Interferometry: Two Case Studies*, LPHYS'23 - to appear soon in the proceedings (2024)